Property Elicitation on Imprecise Probabilities

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Expected Loss Minimization

$$\arg\min_{\theta \in \mathbb{R}^p} \mathbb{E}_{Z \sim P}[\ell(\theta, Z)].$$

- (Asymptotic) Empirical Risk Minimization (Machine Learning)
- Asymptotic M-Estimation
- Property Elicitation

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Property Elicitation

A *property* $f : \Delta(\mathcal{Z}) \to \mathbb{R}^p$ is *elicitable* if there exists a loss ℓ such that

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E.g.,

$$\text{mean} = \arg\min_{\theta \in \mathbb{R}} \mathbb{E}_{Z \sim P}[(\theta - Z)^2].$$

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- Illustrative examples, connections to Γ-maximin and many exciting open problems;
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